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Distribution of the Japanese posted land price and the generalized entropy

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Abstract. The land price can be considered as the economic data which reflects the interactive activity of markets (land market). Since the cumulative distributions often exhibit power law behavior in a number of complex systems, we attempted to investigate the posted land prices available in Japan for the most recent six years of the recession period after the bubble economy. They also seem to be in this category within a certain range. We observe that the generalized exponential function with one parameter which has been frequently used in generalized thermostatistics serving to track the annual change of the distributions.

PACS. 05.90.+m Other topics in statistical physics, thermodynamics, and nonlinear dynamical systems – 89.90.+n Other topics in areas of applied and interdisciplinary physics – 89.75.Da Systems obeying scaling laws

1 Introduction

Stock prices and exchange rates have come to be tracked as econophysics problems, which reflect the complex dynamics of markets with many participants (traders). In market dynamics, traders interact with each other in a highly nonlinear manner. However a number of statistics for the available empirical data show some interesting features which are worth analyzing in terms of statistical mechanics. For example, the power law behaviors appearing in the cumulative distribution of returns would be the most striking property. Since power laws have been found in many different fields by Pareto [1], Mandelbrot [2] and Zipf [3] among others, and we can still find them in many areas. Among them we can cite recent examples of these in the cumulative distributions from the magnitude of earthquakes [4], to the income distribution of companies [5,6] and even further to human sexual contacts [7]. These power law distributions can be attributed to the result of highly nonlinear interactions among the constituent elements.

The same conjuncture seems to apply to the case of land prices because land is traded under the market mechanism. We know that the bubble economy induces a property bubble. In fact, when Japan had the economic boom in late 1980's, land had been the target of money gambling. As a matter of fact, the average land price in the Tokyo area (including the four neighboring prefectures) soared from 158,800 yen in 1980 to 1155,500 yen in 1991 [8]. To predict the movement of the land prices, there are some models [9]. However, these conventional models fail to explain the short-term Japanese land prices as well as the ones for overall categories (residential, commercial area etc.) [9]. Moreover the empirical analysis incorporating the recent data of the contraction phase is still lacking, although there was an attempt to model the urban land market in the bubble era [10].

In the spirit of 'let the data speak', a phenomenological analysis of stock market data is now in vogue and scaling behavior has been reported in numerous distributions [11]. In this paper, we present empirical studies on the posted land prices in Japan and would like to add another piece of evidence which is worth considering as an econophysics problem. Usually the decay in many distributions is approximated by a single power law. However we report an example (land prices) that cannot fit the distribution with a single power law exponent. Previously, Roehner and Sornette investigated the real estate market in Paris [12]. They found that correlation between the number of transactions and the land price per square meter during 1982−1991 bull market was 0.95. Moreover they also found that the same was true for the relation between sales of apartments and prices of land per square meter, except for the existence of a delay. From their implications we consider that the statistics of land prices in Japan also can provide some useful indications for the purpose of understanding of the market dynamics.

On the other hand, the search for alternative statistics has been attempted to give a reasonable fit to the real data. Among them, explanations of the power law scaling in cumulative distributions with the so-called q-exponential function, which arises from the nonextensive

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entropy and its associated statistical mechanics [13], cover citations of scientific papers [14], urban agglomeration [15], and sparseness time intervals on the Internet [16]. In addition a modeling of the dynamics in financial markets from nonextensive thermostatistics is proposed in reference [17]. We shall add another example to them, which can be useful in practice to view the data. In the next section we describe the content of our data and present the results.

2 Empirical results for land price data

According to the white paper on the official land price system in Japan, the official announcement of land prices provides a barometer for transaction prices of ordinary land as well as standards for the estimation of the purchase prices for land intended for public works projects. Since the information about the shape of the land, local circumstances, distance from stations and the maintenance status of electricity or sewage systems are shown in the official announcement on land prices, it also gives an indication for an evaluation of the taxable amount of inherited properties or fixed assets valuation. Furthermore it can be exploited to estimate the current quotation in corporate accounting. The Land Appraisal Committee publishes the nationwide posted land prices once a year, which are intended to be used to form land price estimations in order to regulate land transactions based on the Act for Planning the Utilization of the National Land [18]. The Land Appraisal Committee is a specialized agency established within the Ministry of Land, Infrastructure and Transport. In addition, the land price research of the prefectural and city governments is conducted by prefectural governors once a year against the residential and commercial lands. We call the land that is used to evaluate its price as the average land. Here, the term 'average' does not imply 'mean'. Based on the appraisal reports from more than 2000 real estate appraisers, the governments check and arrange them in order to publish officially on the reference date (July 1st).

In view of these procedures, the land price data totally contrast with the data of high frequency stock markets and foreign exchange rates. Therefore the land price data has cumulative characteristics of economic trends of the year, whereas the high frequency data immediately incorporates the various information into the formation of prices. We also note another essential difference between the stock price data and the land price data. That is, in the stock price data, we can obtain time series data of each brand, whereas we can not extract it for the land price. Since the evaluation of land prices is not necessarily compared with the same average land in consecutive years, tracing the variations of the same place as a time series is practically very difficult.

The data we analyzed was taken from the official web site of the Ministry of Land, Infrastructure and Transport [18]. Table 1 shows the number of the average residential and commercial lands in our data. The average lands are selected by the Land Appraisal Committee as the representative local land reflecting regional characteristics. The land price data for each year contains more than $30,000$ places in total. Land is mainly categorized into two groups (the urbanization promotion area and the urban regulating area) by the city-planning law. The urban promotion area is appointed land where should be preferential promotion of the urbanization and is designed to prevent them from unregulated developments for the sake of better city formations. The urban regulating area, on the other hand, is forbidden to develop.

Table 1. The breakdown of the data set (the number of the average residential and commercial lands).

Year	I	II	III	Total number of lands
1995	24,465	1,697	3,838	30,000
1996	24,444	1,699	3,857	30,000
1997	24,693	1,697	3,910	30,300
1998	24,803	1,697	4,100	30,600
1999	24,964	1,693	4,143	30,800
2000	25,144	1,690	4,166	31,000

Category I: The urbanization promotion area. The residential districts, prospective housing lands, commercial lands, quasi industrial districts and industrial districts are included in this category.
Category II: The urban regulating area. This area consists of housing lands and forestlands.
Category III: The other city [town] planning area. This area falls into the residential districts and commercial lands.

In our data, we obviously observe that the land prices of inner-city districts are higher than those of the other areas for all years. This fact can be easily attributed to the fact that an understanding of the land prices based on the land utilization is equivalent to viewing its utility value. We show the land price distributions for data from 1995 and 2000 in Figure 1. The change in slope is not smooth but breaks at over 200 thousand yen, indicating that there are two scaling classes in the decaying region. This indicates that the corresponding cumulative distributions also have two linear regions on a log-log scale. We confirmed a similar propencity in distributions also for other years in our data. What we found in the original data is the fact that the lands in the central districts of major cities have larger decrease rates than those of other areas. Moreover the number of places with a large decrease considerably differs from year to year.

In general, the land price change in a certain region reflects the local economic conditions (the balance of supply and demand), however at the same time, it can be considered a consequence of a global trend. Figure 2 shows distributions of the annual rate of change observed in the 6-year period 1995−2000 with regard to the mean price change $\langle p_i \rangle$(%) over the year. The curves generally collapse onto the same straight lines, indicating the exponential decay from the peak. However, these shapes are highly *asymmetric*. We observe that the asymmetry or skewness of the distribution varies by region, year, and use. The figure is reminiscent of the distribution of the annual growth rates of the GDP (gross domestic product) for countries [19], in

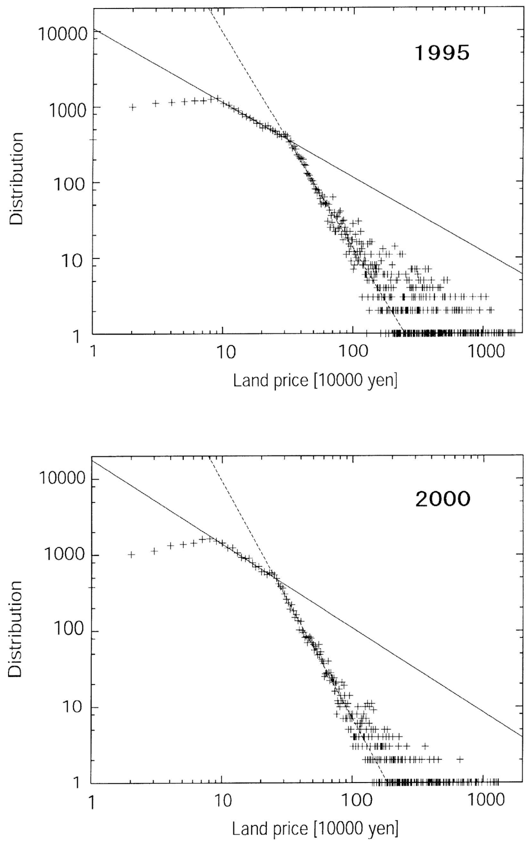

Fig. 1. The semilog plot of the histogram of the land price per square meter in logarithmic bins. The data points are taken from the nationwide investigation in Japan from 1995 to 2000. Two distinct regions which can be well approximated by straight lines are seen. The slopes of the fitted lines are listed in Table 2 for the first and the second region respectively. The first region is in the range of 80 000 yen to around 200 000 yen and follows the second region extending up to near one million yen. We observe that there exists a hump between the two regions. The shapes of distributions were similar for all years in this period, that is, the slopes of the first region take around −2.0 and the second one around −7.0.

which the distributions peaked at zero annual growth rate take *symmetric* tent shapes on a semilog plot. The qualitative comparison between both empirical results would be interesting, however, here we just refer to the fact that the symmetry in the distributions may be a crucial difference. Furthermore, this situation (i.e., the collapse onto a single curve) is not expected for the data of other years. Because the period under study corresponds to the contraction phase of the Japanese economy and the rates of decrease in the mean land prices for the period can be taken as generally constant, while this does not apply for the other periods. Therefore the distributions for the period towards the bubble in the 1980's or for the period of high economic growth of the early 1970's are expected to be totally different from the present shapes of the distribution [20].

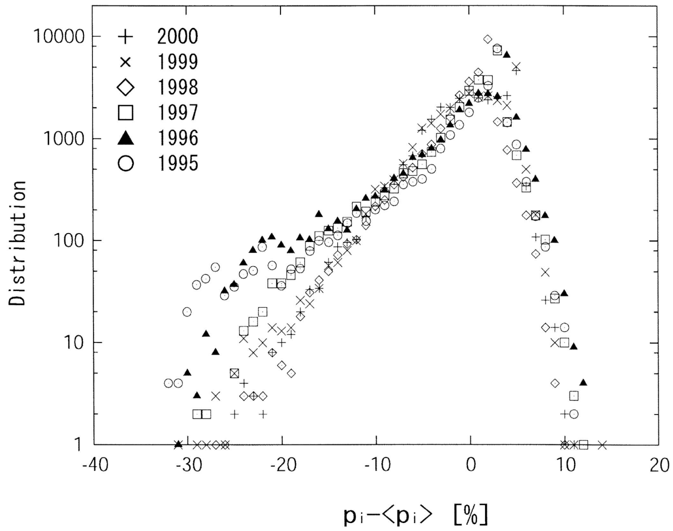

Fig. 2. The distributions of the year-on-year percentage changes of land prices. The tent shape is highly asymmetric, indicating that the land prices, overall, showed a tendency to decrease during this period. We can see the exponential decay for the positive part (increase), whereas the deviation from the straight lines can be seen in the tails of the negative part (decrease) for some years. Namely, the wings on the left side are fatter than is predicted by an exponential dependence.

Table 2. The fitted exponents of the land price distributions.

Year	1995	1996	1997	1998	1999	2000
Region I	−2.19	−2.16	−2.36	−2.32	−2.47	−2.55
Region II	−6.48	−6.56	−6.79	−6.95	−7.07	−7.22

3 The cumulative distribution and the generalized entropy

In connection with the generalized entropy, we are concerned with fitting the normalized cumulative distribution of the land price $P(>x)$ by a function which has been frequently used in generalized thermostatistics. As an example, the MaxEnt approach for the Tsallis entropy shows that the equilibrium distribution function p_i is given in the following form, when we adopt the escort average of quantities $\{E_i\}$ as a statistical mean:

$$p_i = \frac{e_q^{-\beta(E_i-U_q)/c_q}}{Z_q} \tag{1}$$

where q is a real number and the function e_q^x is known in the context of nonextensive thermostatistics [13] as the q-exponential function which reduces to the usual exponential one when $q \to 1$ [21]. Z_q is the generalized partition function

$$Z_q = \sum_i e_q^{-\beta(E_i-U_q)/c_q}, \tag{2}$$

where U_q denotes the escort average of the energy E_i with respect to p_i and $c_q = \sum_i (p_i)^q$. β is a Lagrange multiplier associated with the energy constraint. Here we focus our attention on the curvature of the q-exponential function, which varies with the value of q. We consider that the normalized cumulative distribution takes the following form,

$$P(> x) = e_q^{-ax} \equiv [1 - (1-q)ax]^{\frac{1}{1-q}}. \quad (3)$$

Note that equation (3) arises when we construct the *escort* probability from the original distribution $p(x)$ such that $P_q(x) = p^q(x)/\int p^q(y)dy$ [22] and when we take the cumulative distribution as $P(> x) = \int_x^{x_{max}} P_q(y)dy$, where x_{max} is $1/(1-q)$ if $0 < q < 1$ and infinity if $q \geq 1$ [16]. When the distribution itself takes the form of a q-exponential function, it is related to the conventional Zipf-Mandelbrot law as pointed out in references [15,23]. Moreover equation (3) resembles the Pareto distribution for tails i.e., $P(> x) = cX^{-\alpha}$ by the change of variables $X = c^{1/\alpha}(1 - a/\alpha x)$ and $\alpha = 1/(q-1)$. In addition to this, there may be several functions with one parameter which converge to the usual exponential one. In another form, we may regard the cumulative distribution as

$$P(> x) = e_\kappa(-ax) = \left[-\kappa ax + \sqrt{1+\kappa^2 a^2 x^2}\right]^{\frac{1}{\kappa}},$$

where κ is a real number varying from -1 to 1 [24]. This *κ-exponential* function which recovers the usual exponential one in the limit $\kappa \to 0$ was defined in κ deformed thermostatistics [24]. At this point, there is no criterion for choosing the extended exponential function originating from the associated generalized thermostatistics, however, we assume here that the underlying statistics is dominated by the q-exponential function. We think that the translation of the value of κ into the value of q is possible by fitting the same data. In Figure 3, we compare the cumulative distributions of the absolute value of the annual percentage change of land prices for two different years i.e., 1995 and 2000. The qualitative difference in the distribution between the two years is clearly seen, in which we consider that the shape of the distributions is quantified by the parameter q for a certain range. We show the results of the nonlinear fits with the function e_q^{-ax} to the normalized cumulative distributions in Table 3. Although a departure from the regression lines is seen for larger annual changes of land prices for all years under study (for over 20%), the determination of the coefficients for the fits are reasonable within our range of fits. At present we benefit from the use of the q-exponential function for the capability of fitting without neglecting the shapes of the heads of the distributions. Here, it is worth noting that in the physical system within nonextensive statistical mechanics, we often encounter examples of systems with q values having either $q > 1$ or $q < 1$. The land price data of 1995 and 2000 are just such cases. Moreover the value of q steadily decreases in this period under study. One finds that the shape of the distribution passed through the exponential ($q = 1$) between 1998 and 1999. At this point, we do not have an appropriate explanation for the motion of the q values, however, the change of index phenomenologically indicates that some sort of structural changes had happened in the land market in Japan. The fact that the tendency for departure from the regression lines for the large annual changes in the distributions (larger than about 20%) for all years seems to indicate that the price formation process with larger percentage changes differ fundamentally from ones with the smaller changes. To confirm this conjecture we would need to study the microscopic dynamical model and compare it with the data, however, it exceeds the scope of our present paper.

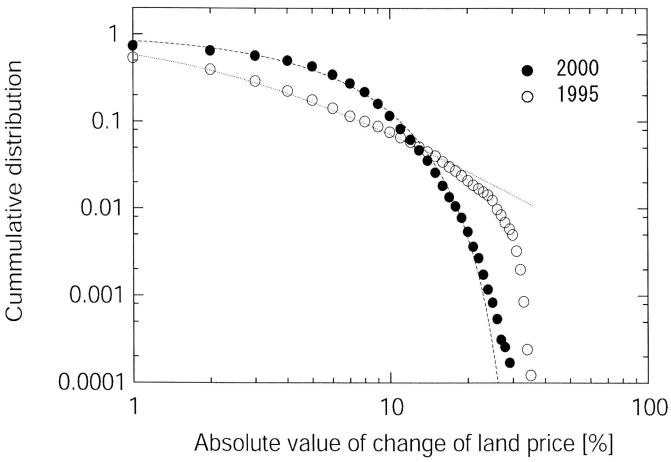

Fig. 3. The cumulative distribution of the absolute value of the year-on-year percentage changes of land prices in 1995 and 2000, together with the nonlinear fits (regression lines with $q = 1.590$ and $q = 0.8193$ for 1995 and 2000 respectively). Note that the regression lines explain the smaller changes in land prices rather than tail parts.

Table 3. The nonlinear fit to the normalized cumulative distributions with e_q^{-ax}.

Year	1995	1996	1997	1998	1999	2000
q	1.590	1.372	1.360	1.285	0.9228	0.8193
a	0.6366	0.3536	0.4698	0.5096	0.1974	0.1715
R^2	0.9930	0.9918	0.9954	0.9951	0.9890	0.9904

4 Summary

We have first analyzed the Japanese posted land price data from 1995 to 2000. In this period under study, the Japanese economy has continued to slowdown after the collapse of the asset-inflation bubble. This tendency is still going on without any effective explanations. Our important findings are compiled into two facts. That is, there are two power law scaling regions in the distribution of the land prices for all years analyzed. Furthermore we have found that the cumulative distributions of the absolute change of land prices can be well described by the power law and are fitted with a generalized exponential function i.e., the q-exponential distribution at least for a certain range. In this sense the q value serves to quantify the aging of the distributions. In general, the price formation of the land market is terribly complex and multifaceted [25]. The factors involved are social, economical, administrative and regional environments or land utilization. However, building a reasonable model which incorporates these factors is difficult task because of the lack of detailed information. The relation between the land prices and the change of rate of stocks is now regarded as a central issue in the political strategy of many countries [26]. In this sense our empirical findings may provide elements for studying the land price formation mechanism in terms of econophysics.

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Note added

The author, on his revision process, became aware of a reference [27], which had dealt with the same data for similar analyses.

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